Longitudinal Analysis of Prices of Real Estate Rental

Israel José dos Santos Felipe
Universidade Federal de Ouro Preto
E-mail: israeljfelipe@gmail.com

Ivair Silva Ramos
Universidade Federal de Ouro Preto
E-mail: ivairest@gmail.com

Thiago de Sousa Barros
Universidade Federal de Ouro Preto
E-mail: tsousabarros@gmail.com

Diego Carvalho do Nascimento
USP/UFSCar
E-mail: dnstata@gmail.com

Abstract

Purpose
The aim of this study was to analyze the real estate market (residential and commercial) from Mariana’s city Minas Gerais State. The motivation for choosing this city lies in its importance to the economy of the Minas Gerais State.

Design/methodology/approach
We conduct a statistical analysis adopting the longitudinal models (fixed and random effects) in 147 properties. The collection of data covered the commercial and residential real estate prices over a period of ten years.

Findings
The statistical modeling results showed a presence of random variations higher than the typical prices considered in the real estate market. Estimates have also suggested that commercial real estate prices are on average higher than residential ones.

Research limitations/implications
The analysis could cover real estate prices (residential and commercial) at different times of analysis, but due to the limited expansion of the database, we are unable to conduct more comprehensive statistical analyzes.

Originality/value
Until this present date, no other research of this nature has been detected, at least in the Brazilian market, which analyzed the real estate market price dynamics by the segmentation as proposed in this study. We believe that based on this work, other researchers will develop similar studies seeking to describe the dynamics of real estate market prices in others important Brazilian’ cities. Such information may be relevant to more effective urban planning and delimitation of more efficient public housing policies.

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INTRODUCTION

The Brazilian real estate market has undergone several changes in recent decades, following the creation of the Residence Investment Fund and the stabilization generated by the Real Plan, and by the policies implemented by the government as the “Minha Casa Minha Vida” program. In the field of study, the city of Mariana, according to IBGE (2010) in recent decades suffered a strong population growth, higher than other regions of the Minas Gerais (MG) State.

This population growth can be characterized by the strong mining activity in the region, which is characterized by the presence of basically two companies, namely: Vale and Samarco. Mineral activities carried out by these firms require a lot of manpower, resulting in job postings generating migration to the city and also due to the presence of the Federal University of Ouro Preto (UFOP), which was implemented in Mariana, in 2008, the Institute of Applied Social Sciences - IASS, having two university campus in the city.

These factors contributed to a substantial increase in GDP. According to the Census (IBGE 2010), in 1999, the Mariana GDP was equivalent to R$ 330,092, and in 2011 the GDP was equivalent to R$ 5,443,576, representing a 1,549% growth. So, just as there was an increase in the supply of housing, as a result of government programs, there was also an increase in the demand due to the population and income growth, which generates therefore increased consumption.

According to Cintra and Farhi (2008) the US housing market for many years was characterized by a low rate of interest, which enabled financing of housing, similar to the Brazilian. But in recent years, the United States experienced a housing crisis caused by excessive mortgages, rising defaults, and devaluation in property prices.

One of the main reasons for this scenario was the sale of assets related to subprime mortgages, or mortgages to individuals who did not have fixed income, characterized by a higher probability of default. Banks used the securitization mechanism to disguise these mortgages using foreclosed homes as collateral. Thus, to encourage people to take these loans was used the progressive interest mechanism in which at first were charged low-interest rates, but increased progressively. The character of the interest comes from the American amortization system (AAS), which are paid only the interest on the loan and is not charged any value in the main borrowed, unlike the other amortization systems in which it is regularly paid interest and
The AAS and other factors led in late 2006, the impossibility of payment of interest by the holders of mortgages, the default was inevitable.

The general decline resulted in a housing crisis as a result of mortgages being used as financial assets, on which banks sold these debts to investors. This occurred as a result of non-payment of mortgages where holders of active agents understood they would receive fewer returns and in an attempt to not suffer losses, they tried to sell the assets in the market. However, several did this practice and resulted in more active supply than demand, leading to devaluation. The speculation generated by the breakdown of banks and insurance companies made a serious outbreak internationally crisis that led to the devaluation of the real estate market, causing a drop in property prices.

The US housing crisis of 2008 and the increase in prices of Brazilian real estate according to Zornoff (2012) cause questionings about the support of these prices, since the present speculation in real estate make the price more volatile. The term volatility may be expressed by the price change, i.e. its buoyancy in certain periods. This may be caused by the amount of information present in the market, supply and demand of assets, market performance, among others. According to Jubert et al. (2008) in the financial market can be observed various periods of high volatility or low volatility, this depends on whether the price change is positive or not.

Thus, the importance of the study of volatility occurs due to the need to understand the past prices and hence predict the future price at which it can identify if it is profitable to invest in particular assets. This way, through this study, will be possible to measure the volatility of prices of rents in the city of Mariana, MG.

This study investigated the changes in the prices of rents of real estate (residential and commercial) of Mariana-MG through statistical modeling (panel data analysis). Such research is important for the formulation of housing policies of Mariana-MG region since according to exploratory studies in the region (Magazine “Dois Pontos”, 2013), there is a strong avoidance of residents to other cities because of the cost of living, especially the rent.

As main results, it was observed that: the projections for the rental prices deserve to be treated for each property individually, because the random effects from individual to individual is the main factor that explains the overall variability of the price series; the final model adopted strengthened the suspicion that the global average price of rents has a tendency to increase over time; it is estimated that no correlation exists between the prices of the levels observed in 2004
and the rate at which will increase over time, and; it is estimated that the average prices of commercial real estate rents are around 1.2 times higher than the average prices of residential.

Finally, this paper is divided into five parts. The next section presents a review of the literature on housing market studies, economic variables, price volatility and statistical analysis of rental prices. Section 3 presents the general methodological aspects of this research. A discussion of the main results of statistical analysis of the series of rental prices is presented in section 4 and finally, Section 5 presents a brief discussion of the main findings and conclusions of this research.

EMPIRICAL LITERATURE REVIEW

Over the years some authors seek to analyze the housing market, like González (1993), Santos and Cruz (2000), among others. González (1993) points out that the housing market is determined by the stock, due to the durability of buildings, in which there is a direct relationship between the prices of new homes and those already on the market. As the supply of new homes tend to be small relative to the already existing, the supply of the housing market is inelastic, which means that price changes are caused by demand.

Santos and Cruz (2000) argue that the housing market has distinct features compared to other markets, this occurs as a result of that housing is a good "meritorious", i.e. a good that can be produced, but with high cost, resulting in the inability to access of some individuals. Accordingly, Menezes et al. (2007) point out that the cost of rent is also given by supply and demand, due to the low price elasticity of supply of land. Thus, an increase in demand will result in a more than proportional increase in the prices of rents. The rise of the rental price causes a change in the cost of tertiary and industrial activities, which causes an increase in final prices of goods.

Following to yet Menezes et al. (2007) there is a relationship between urban and rural properties in which industries and individuals compete for space. The increase caused by the prices of rents reduces the competitiveness of rural areas since residents of urban areas can make bigger deals. Rent is considered a synthesizer cost of living because the elevation of the same leads to an increase in prices throughout the economy.

Faced with the volatility of the housing market, the choices of individuals buy real estate, depends not only on its market price. Corresponding Arraes e Sousa Filho (2008) discuss that the choice of the property by the consumer depends on other three characteristics: i) physical
attributes of the asset; ii) neighborhood and iii) environmental characteristics of the property. Therefore, in the price analysis, not only the physical characteristics are important, but also the effects of externalities.

This variation in the price of rents occurring in the real estate market can be characterized by existing volatility due to structural changes that may occur in a given period of time. There is no statistical evidence regarding the instability in the housing market and, due to the lack of evidence, there are two obstacles: "the availability of reliable data and methodological issue" (KANDIR, 1983, p.16).

With respect to data availability, there are several difficulties in obtaining these, such as the heterogeneity of the products and the fact that the inputs used to build are also used in other sectors. For González (1993) other factors affecting the data is the existence of problems such as durability, immobility and high cost. Hui and Zheng (2012) disagree with this argument, based on the premise that one should analyze the dynamics of house price with economic variables such as income, population, in which was found heterogeneity over time between the price and other variables.

Miller and Peng (2006) use the GARCH and VAR model to analyze the temporal variation in household spending and changes in the economy. According to Hui and Zheng (2012), using models present value and error correction models some researchers tested the existence of an implicit relationship between housing prices and rents. This was not possible given that normally uses to income as noise and the relationship between price and rents are usually not constant.

Explained by Kandir (1983) all sectors submitted to capital are unstable nature, in the case of real estate, this tends to be higher since it uses the current production of housing. The author points out that factors such as the difficulty of programming, control, and execution of constructive activities, have an important role in instability in the housing market. Another factor that creates instability in the market is the high mobility of capital invested in the sector.

Bonfim (2005) asserts that speculation does not occur simply and isolation and is not caused only by the owner of the property, this is given by the complexity of the urban environment that is always in the process of transformation. According to Resende (2012) and Zmitrowicz and Neto (1997) real estate speculation generates appreciation or depreciation in the occurrence of these locations, depending on the speculative capital interest. This same author also
considers the role of soil regulator and infrastructure provider, the state, in which the actions taken by the state tends to influence future decisions in real estate. Holders of speculative capital seek partnerships with public authorities, to new business strategies and real estate productions. Therefore, speculation is of essential interest of the owners Zmitrowicz and Neto (1997) and Resende (2012).

Real estate speculation is related to variability or instability of the market, in which agents stockpile assets in the belief that there is an increase in the prices, in order to receive profit. Says Kandir (1983) so that there is speculation it is necessary that the active cause in individuals the sense that at some point the offer will not adequately respond to demand, leading to rising prices, i.e. the asset valuation, requiring that it possesses a specific market to have liquidity, which will enable their sale at an appropriate time.

In this way, as Kandir (1983), speculation is caused by demand, since a variation in demand generates a small rise in prices, which encourages speculators to increase their stock of goods, hoping that the price of this increase. The speculation mechanism is a vicious circle, as the first signs of speculation generate an increase in demand, given that the supply is fixed, occurs again rising prices. Speculators believing that the assets are valued carry out the purchase of more units, shifting demand again.

Saboya (2008) cited by Resende (2012) adds that there is not a consensus that defines the speculation, as this may indicate the mobility of construction and sale of buildings, or for the retention of urban real estate holdings for future negotiations, given the expected appreciation of them. According to Silva Junior (2007) speculate is to keep something in the hope that after the negotiations it can be sold at a higher value than the previous one, and are the external factors that cause occurring valuations.

The price appreciation in the real estate market can be seen in many countries as the United States, which resulted in the real estate crisis - 2008 subprime in European countries like Spain Ireland and Portugal which had in recent years a huge appreciation in property prices. Similar to Brazil, it was the housing market that was heated due to the existence of incentives for real estate credit and low-interest rates, in the confidence that the housing market would continue booming. (DAGOSTINI, 2010).

Cagnin and Cintra (2009) confer that the American economy went through a process of appreciation of property prices, in which individuals were betting that real estate would value
increasingly, being possible that later were held the mortgages payment comfortably, however, these have lost value market, generating the trigger of the 2008 crisis. In the European scenario, the crisis affected mainly the countries of southern Europe, these that have an economy with more debt and fiscal problems.

The research developed by Ibanez and Pennington-Cross (2013) investigate commercial property rent dynamics for 34 large metropolitan areas in the US, for the series 1999-2009. The results indicate is has substantial heterogeneity in both the long and short run responses to changing demand and supply conditions. Furthermore, the office market is slowest to adjust back towards equilibrium while industrial and flex markets adjust back to the long run equilibrium very quickly.

In a recent article, White and Ke (2014) considering the accelerated development of office markets in Shanghai in the last twenty years and consequences on the office submarkets: Puxi and Pudong. This raises the issue as to whether the Shanghai office market can be viewed as a homogeneous entity or whether there is imperfect substitutability across office locations within the city. But, the paper not find no interaction between the two submarkets and find no evidence of lead-lag relationships between the two submarkets. Additionally, the tests for convergence in rental performance between the two submarkets no indicate convergence.

Analyzing the Brazilian scenario, it is noticed a significant increase in the IPCA, especially in 2013. Characterized by rising rents prices, services such as Condo prices. Where in December 2012 the prices of residential rents, presented in index 1.56% settling in second place of the individual impacts. And housing costs were discharged from 0.63% at the end of December. Agostini (2010) points out that in the period 2003-2010 the National Index of Construction (NIC) increased 63%, as the average salary of employed individuals increased only 22.65%, a clear example that the housing market inflated its prices, characterizing the beginning of a real estate bubble. In the period from 2010 to 2012 occurred in Brazil a variation of about 43% in real estate sales prices. The city of Rio de Janeiro with the greatest variation, then the city of São Paulo, showed price change of real estate 58.86% and 43.16%, respectively. At the same time, the stock of housing credit varied 130%.

According to Mendonça and Sachsida (2012) the price stability, the fall in interest rates, the expansion of real estate credit, public programs, modernization and revitalization of cities for the World Cup and the Olympics in Rio in 2016, are factors that explain this intense housing
boom in recent years. In which the city Rio de Janeiro, was the one with most appreciation, characterizing the current speculation in Brazil.

**METHODS**

*Data base*

For the longitudinal analysis of the data were collected prices of rents in the city of Mariana, commercial and residential in this location the data analyzed refer to the ten-year period, ranging between 2004 and 2014, data were collected from monthly data rents. The survey was conducted with Mariana MG Real Estate, totaling 147 buildings covering the entire city of Mariana, Minas Gerais. The properties are further divided into two groups: commercial real estate and residential real estate. After collecting the data, they were tabulated divided into commercial or residential for subsequent analysis.

*Statistical Methods*

As previously mentioned, this research seeks to model the volatility of the prices of rents in Mariana, Minas Gerais municipality. Therefore, one of the most appropriate statistical methods is the panel data method, starting from the premise of a temporal analysis.

For Cameron and Trivedi (2009) the panel data refer to a group of observations in cross section for many periods. In consequence of these features, the panel data have several advantages. These authors also emphasize that the biggest advantage of using panel data is the increased estimation accuracy by increasing the observations as a result of combinations of data of individuals with various periods of time. And also the possibility of the estimation to be consistent as a result of the models include the individual unobserved heterogeneity caused by omitted variables.

According to Baltagi (2005), there are other benefits in the use of panel data regressions, such as i) Control of heterogeneity, that in the absence thereof may lead to biased results; ii) Increased variability, less collinearity between variables; iii) Elimination of bias due to aggregation of data at different levels that should be analyzed.

Marques (2000) explains that the advantage in the use of panel data is the individual heterogeneity ratio. In order that the panel data suggest the existence of differentiating characteristic of individuals understood as a basic statistical unit. The author also highlights other
advantages in panel data model, such as a greater amount of information, higher data variability, less collinearity between variables, the greater number degree of freedom and greater efficiency in the estimation.

Cameron and Trivedi (2009) point out that there is a branch of literature that seeks to explore the panel data to control the unobserved heterogeneity. Existing unobserved variables, and not correlated with other explanatory variables in the model, these will be included in the error term. However, having some correlation will result in the problem of omitted variable bias, affecting the regression estimates. According to Wooldridge (2010), the main motivation for using panel data is the problem of existence of omitted variables in a scenario where the panel can be used in the regression consistent estimators, even in the event of unobserved variables. Longitudinal data enable control of unobservable variables, mainly occurred through two models: the fixed effect model and random effects model.

**Fixed and random effects models**

In the fixed effects model it is possible that the unobserved variables are correlated with other explanatory variables were not observed effects refer to the individual characteristics not observed and constant in time. In order that the none observed features are present in the ci fixed effects regression, such that:

\[ y_{it} = X_{it} \beta + c_i + \epsilon_{it} \]

Where \( y \) is the dependent variable, \( X \) is the explanatory variables, \( \beta \) coefficients to be estimated, \( c \) refers to the non-observed characteristics and the \( \epsilon \) random error term. While \( i \) and \( t \) represent the individual and time. Thus, the identification hypothesis requires that \( E(\epsilon_{it} | c_i, X_t) = 0 \), i.e. the error is not correlated with the explanatory variables or the omitted variables, and still means that the unobserved variables have no correlation with the explanatory variables.

The fixed effects model controls the effects of the omitted variables that vary between individuals and remains constant over time explained Cameron and Trivedi (2009). Assuming that the intercept varies from one individual to another, but is constant over time, may also vary its constancy over a period of time. The authors emphasize another model assumption considering the intercept as fixed and unknown parameter in other to capture the differences between individuals who are in the sample, i.e. the model inferences represent only the data of individuals in the sample, not the population as a whole.
The same authors point out that the random effects model has the same assumptions of the fixed effects model, i.e., the intercept varies from one individual to another, but not over time, where the difference between the two models refers to the treatment of the intercept. The variable effect model treats the intercept as random variables, considering that the individuals in the database are random samples from a larger population.

Corroborating, Wooldridge (2010) argues that the main reason to decide which model to use is the unobserved effect \( c_i \). The existence of a \( c_i \) uncorrelated with all the explanatory variables, the random effect is the most suitable. Since otherwise, it uses the fixed effects model. Greene (2005) states that to see if there is a correlation between \( c_i \) and the explanatory variables, we use the Hausman test in order to use the null hypotheses and alternatives, indicating whether \( c_i \) is correlated or uncorrelated with other explanatory variables.

**ANALYSIS AND DISCUSSION OF RESULTS**

*Initial analysis of rental prices*

Before beginning the formal statistical modeling, it is essential to make an exploratory and descriptive data analysis. It is worth mentioning that six buildings stood out for having excessively higher prices than others. Their prices extrapolated two interquartile distance in relation to the annual price medians, three from the residential and three from the commercial.

Overall, prices for commercial real estate are higher than residential, and this feature is realized by any of the descriptive statistics presented in Table 1. The boxplot graphs shown in Figure 1 show the overall behavior of the series over the years. The above outliers’ properties were subtracted from this representation. The graphs of Figures 1 (A) and Figure 1 (B) show the data on the scale of the original price for residential real estate and commercial respectively. It is observed that the variability seems to increase over the years. A logarithmic transformation favored the stabilization of variability from year to year, and this is illustrated in Figures Figure 1 (C) and Figure 1 (D).
Table 1. 
Global descriptive for the real estate prices in the period 2004-2014

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>Minimum</td>
<td>203.2</td>
<td>227.3</td>
<td>231.2</td>
<td>239.1</td>
<td>259.2</td>
<td>280.3</td>
<td>278.4</td>
<td>310.4</td>
<td>324.5</td>
<td>350.2</td>
<td>370.0</td>
</tr>
<tr>
<td>1st Quartile</td>
<td>384.4</td>
<td>430.0</td>
<td>437.5</td>
<td>452.4</td>
<td>490.3</td>
<td>530.3</td>
<td>544.2</td>
<td>618.8</td>
<td>646.8</td>
<td>698.0</td>
<td>737.5</td>
</tr>
<tr>
<td>Median</td>
<td>549.4</td>
<td>614.3</td>
<td>646.3</td>
<td>646.3</td>
<td>700.4</td>
<td>757.5</td>
<td>752.5</td>
<td>839.0</td>
<td>877.0</td>
<td>946.4</td>
<td>1000.0</td>
</tr>
<tr>
<td>Mean</td>
<td>688.1</td>
<td>769.6</td>
<td>784.7</td>
<td>812.5</td>
<td>878.5</td>
<td>951.3</td>
<td>950.8</td>
<td>1055.1</td>
<td>1105.1</td>
<td>1192.3</td>
<td>1246.2</td>
</tr>
<tr>
<td>3rd Quartile</td>
<td>823.7</td>
<td>921.4</td>
<td>937.5</td>
<td>980.6</td>
<td>1050.6</td>
<td>1136.3</td>
<td>1128.7</td>
<td>1258.5</td>
<td>1315.5</td>
<td>1419.6</td>
<td>1500.0</td>
</tr>
<tr>
<td>Maximum</td>
<td>3624.0</td>
<td>4054.2</td>
<td>4124.8</td>
<td>4265.2</td>
<td>4999.6</td>
<td>4966.2</td>
<td>5537.3</td>
<td>5788.3</td>
<td>6246.4</td>
<td>6600.0</td>
<td>7000.0</td>
</tr>
</tbody>
</table>

Residential Median |
| Mean     | 879.1| 983.3| 1016.0| 1060.2| 1143.2| 1241.5| 1261.7| 1394.8| 1461.5| 1572.6| 1713.7|
| 3rd Quartile | 1085.1| 1214.3| 1292.9| 1292.9| 1400.9| 1515.0| 1504.9| 1678.0| 1754.0| 1892.9| 2000.0|
| Maximum  | 4392.8| 4914.2| 4999.8| 5170.0| 5603.4| 6060.1| 6019.7| 6712.2| 7016.1| 7571.4| 8000.0|

Source: Research data.

Figure 1. 
Quartile and extreme values in property prices in the period 2004-2014

Source: Research data.

The price series collected for this study shows a typical configuration, which induces a longitudinal analysis for mixed effect models. Figure 2 shows the profiles of the 100 price series
for residential properties (left panel) and the 47 series of prices for commercial real estate (right panel). In order to favor a set of clearer view series, Figure 2 was built on the scale of the Neperian logarithm of the price. There appears to be considerable variability in price between real estate. Comparatively, intra-property variability is not excessive, indicating that the major sources of variation are probably derived from the time of variable and random effects present in the overall average components and temporal trend. It should be noted that intra property variability does not seem to change from one property to another, and this is important because the validity of statistical inferences during modeling will depend on the satisfaction of homoskedasticity assumption also among real estate.

The positive trend of fixed effect is also evident, but does not seem to correlate with the overall average term (straight line intercept) because, regardless of the level at which each series began its career in 2004, the inclination of the apparent line that is formed over time is similar to the different time series. Also it doesn’t seem to have a significant change in slope of one property price curves for each other. But prices for commercial real estate appear to have a greater variability of the property type to property.

Figure 2.
Profiles of the series of real estate price for the period 2004-2014

Source: Research data.
Statistical Inference

The evidence found in the descriptive analysis suggests the existence of two random effects, one for the global average term (intercept) and one for the time trend term. Besides that, it is reasonable to consider that there are three fixed effects for each property associated with the global average term, the slope of the time trend and the type of property (residential or commercial). Thus, a mixed model to estimate these five (05) main structures that potentially explain the variability of price logarithm was adjusted. Random effects for a given property are deviations in intercept and slope of that property in relation to these same characteristics as the real estate population.

The trend term in the model will be represented by the sequence of integers from 1 to 11, wherein the number 1 represents the 2004 and 2005 represent 2, and so on. Types of properties will be represented by a dichotomous variable that takes the value 0 for residential real estate and the value 1 for the commercials.

Initially, two models were adjusted, here labeled by FM1 and FM2. The first does not consider the fixed effect of property type, and the second includes such purpose. See the models:

Model FM1:
\[ Y_{ij} | c_i \sim \text{NORMAL}(\mu_{ij}, \theta), \quad i = 1, \ldots, 147 \text{ (BUILDINGS)} \quad e \quad j = 1, \ldots, 11 \text{ (YEAR)} \]
\[ \mu_{ij} = \beta_0 + \beta_1 Year + c_i, \quad c_i \sim N(0, \Sigma_i) \]

Model FM2:
\[ Y_{ij} | c_i \sim \text{NORMAL}(\mu_{ij}, \theta), \quad i = 1, \ldots, 147 \text{ (BUILDINGS)} \quad e \quad j = 1, \ldots, 11 \text{ (YEAR)} \]
\[ \mu_{ij} = \beta_0 + \beta_1 Year + \beta_2 Type + c_i, \quad c_i \sim N(0, \Sigma_i) \]

Estimates for the main terms of the mixed effects model FM1 are presented in Table 1. These estimates were obtained by the use of lmer package in R. Results for the FM2 model are shown in Table 2.
Table 1.

**Results for FM1 model**

(fm1 <- lmer(price ~ 1 + Year + (1 + Year | Real state), data))

Mixed Linear Model - REML

price ~ 1 + Year + (1 + Year | Real state)

<table>
<thead>
<tr>
<th>AIC</th>
<th>BIC</th>
<th>LogLik</th>
<th>Deviation</th>
<th>REMLdev</th>
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<td>-5.201</td>
<td>-5.168</td>
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<table>
<thead>
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<th>Group</th>
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<th>Correlation</th>
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</thead>
<tbody>
<tr>
<td>Real state</td>
<td>(intercept)</td>
<td>2.99E+03</td>
<td>0.5466604</td>
<td>-</td>
</tr>
<tr>
<td>Year</td>
<td>-</td>
<td>6.68E-01</td>
<td>0.0081752</td>
<td>0.012</td>
</tr>
<tr>
<td>Residuals</td>
<td>-</td>
<td>8.95E+00</td>
<td>0.0299112</td>
<td>-</td>
</tr>
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</table>

**Fixed effects:**

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<tr>
<th>Estimate</th>
<th>Standard Deviation</th>
<th>Value t</th>
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<tr>
<td>(intercept)</td>
<td>6.4074309</td>
<td>142.02</td>
</tr>
<tr>
<td>Year</td>
<td>0.0590888</td>
<td>82.74</td>
</tr>
</tbody>
</table>

Fixed Effects Correlations:

(Intr)

Ano 0.001

Source: Research Data.

Table 2.

**Results for the FM2 model**

(fm2 <- lmer(price ~ 1 + Year + type + (1 + Year | Real state), data))

Mixed Linear Model - REML

price ~ 1 + Year + type + (1 + Year | Real state)

<table>
<thead>
<tr>
<th>AIC</th>
<th>BIC</th>
<th>LogLik</th>
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<th>REMLdev</th>
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<table>
<thead>
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<th>Variance</th>
<th>Standard Deviation</th>
<th>Correlation</th>
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<td>Real state</td>
<td>(intercept)</td>
<td>2.92E+03</td>
<td>0.5404982</td>
<td>-</td>
</tr>
<tr>
<td>Year</td>
<td>-</td>
<td>6.68E-01</td>
<td>0.0081752</td>
<td>-0.019</td>
</tr>
<tr>
<td>Residuals</td>
<td>-</td>
<td>8.95E+00</td>
<td>0.0299112</td>
<td>-</td>
</tr>
</tbody>
</table>

**Fixed effects:**

<table>
<thead>
<tr>
<th>Estimate</th>
<th>Standard Deviation</th>
<th>Value t</th>
</tr>
</thead>
<tbody>
<tr>
<td>(intercept)</td>
<td>6.3419699</td>
<td>117.28</td>
</tr>
<tr>
<td>Year</td>
<td>0.0590888</td>
<td>82.74</td>
</tr>
<tr>
<td>type</td>
<td>0.2047398</td>
<td>2.14</td>
</tr>
</tbody>
</table>

Fixed Effects Correlations:

(Intr)

Year -0.023

Type -0.565 0.000

Source: Research Data.
Table 3.

**Results of the comparison between FM1 and FM2 models (ANOVA)**

anova(fm1, fm2)

Models:

fm1: price ~ 1 + Year + (1 + Year | Real state);  fm2: price ~ 1 + Year + type + (1 + Year | Real state)

<table>
<thead>
<tr>
<th></th>
<th>Df</th>
<th>AIC</th>
<th>BIC</th>
<th>LogLik</th>
<th>Chisq</th>
<th>Chi Df Pr (&gt;Chisq)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FM1</td>
<td>6</td>
<td>-5217.5</td>
<td>-5185.2</td>
<td>2614.8</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>FM2</td>
<td>7</td>
<td>-5220.0</td>
<td>-5182.3</td>
<td>2617.0</td>
<td>44.473</td>
<td>1 0.03496 *</td>
</tr>
</tbody>
</table>

Codes:

0 '****' 0.001 '***' 0.01 '**' 0.05 '*' 0.1 ' ' 1

Source: Research Data.

In order to check the need to include a fixed effect for type of property in the model, the likelihood ratio test was used to compare models FM1 and FM2. The results of this test are presented in Table 3. The value of the chi-square statistic, 4.4473, is relatively high, corresponding to a p-value of approximately 0.0349, indicating that for any level of significance greater than 0.035, the extra parameter in the model FM2 in relation to FM1 produces an adjustment significantly better.

Another feature noted in the FM2 model is that the correlation between the random effects is extremely low, about -0.019. It had been noticed during the descriptive analysis that does not seem to be a correlation between the global average and the slope of the time trend. So a third model, FM3, was adjusted disregarding the correlation between the random effects of intercept and trend. Table 4 presents the estimates for FM3.

Model FM3:

\[ Y_{ij} | c_i \sim \text{NORMAL}(\mu_{ij}, \theta), \quad i = 1, ..., 147 \text{ (BUILDINGS)} \quad e \quad j = 1, ..., 11 \text{ (YEAR)} \]

\[ \mu_{ij} = \beta_0 + \beta_1 Year + \beta_2 Type + c_i, \quad c_i \sim N(0, \Sigma_i) \]

Table 5 shows the results of the likelihood ratio test for comparison of models FM3 and FM2. The value of the chi-squared statistic equals 0.0453, is quite small, so that the associated p-value is very high, equal to 0.8315, indicating that the extra correlation parameter in the model FM2 in relation to FM3 does not produce an adjust significantly better. Therefore, through the information criterion AIC e BIC, the FM3 model is the best candidate for the final model.
Table 4.

**Results for FM3 model**

(fm3 <- lmer(price ~ 1 + Year + type + (1 + Year|Real state) + (0+Year | Real state), data))

Mixed Linear Model - REML

\[
\text{price} \sim 1 + \text{Year} + \text{type} + (1 + \text{Year} | \text{Real state}) + (0 + \text{Year} | \text{Real state})
\]

<table>
<thead>
<tr>
<th>AIC</th>
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<th>LogLik</th>
<th>Deviation</th>
<th>REMLdev</th>
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</thead>
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<td>-5.202</td>
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<td>2.607</td>
<td>-5.234</td>
<td>-5.214</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Group</th>
<th>Name</th>
<th>Variance</th>
<th>Standard Deviation</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real state</td>
<td>(intercept)</td>
<td>2.92E+03</td>
<td>0.5403965</td>
<td>-</td>
</tr>
<tr>
<td>Year</td>
<td>-</td>
<td>6.68E-01</td>
<td>0.0081735</td>
<td>-</td>
</tr>
<tr>
<td>Residuals</td>
<td>-</td>
<td>8.95E+00</td>
<td>0.0299118</td>
<td>-</td>
</tr>
</tbody>
</table>

Fixed effects:

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>Standard Deviation</th>
<th>Value t</th>
</tr>
</thead>
<tbody>
<tr>
<td>(intercept)</td>
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<td>117.30</td>
</tr>
<tr>
<td>Year</td>
<td>0.059089</td>
<td>0.000714</td>
<td>82.76</td>
</tr>
<tr>
<td>type</td>
<td>0.201368</td>
<td>0.095626</td>
<td>2.11</td>
</tr>
</tbody>
</table>

Fixed Effects Correlations:

(Inter) Year

Year -0.009

type -0.565 0.000

Source: Research Data.

Table 5.

**Results for the comparison between the FM2 and FM3 models**

anova(fm2, fm3)

Models:

fm3: price ~ 1 + Year + type + (1 | Real state) + (0 + Year | Real state)

fm2: price ~ 1 + Year + type + (1 + Year | Real state)

<table>
<thead>
<tr>
<th>DF</th>
<th>AIC</th>
<th>BIC</th>
<th>LogLik</th>
<th>Chisq</th>
<th>Chi</th>
<th>Df Pr (&gt;Chisq)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FM3</td>
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<td>-5221.9</td>
<td>-5189.6</td>
<td>2617.0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>FM2</td>
<td>7</td>
<td>-5220.0</td>
<td>-5182.3</td>
<td>2617.0</td>
<td>0.0453</td>
<td>1</td>
</tr>
</tbody>
</table>

Source: Research Data.
Figure 3.
Waste correlations between the years

Source: Research Data.

Figure 4.
Waste per year

Source: Research Data.
The residue analysis shows that, in general, the correlations (Figure 3) were not significant between the years and therefore, intra-property variability appears to be well-represented by the FM3 model. Figure 4 shows the dispersion of the waste over the years which appear to be randomly dispersed around zero.

**Interpretation of the final shaping**

At this moment, the results of the FM3 model are interpreted, which is hereafter treated as the final model chosen. Confirming the initial suspicion, the arising variation of the random effect term between the buildings, reflected by the estimated standard deviation around 0.5404, is far more significant than the variation brought by the random effects of model slope, for which observed a standard deviation of 0.008. The source of intermediate variation and not explained by the model, the variance of marginal noise term, presented a standard deviation estimate of about 0.029 log units.

The fixed effect for the model intercept is estimated at 6.343 log units. This value can be interpreted as an overall average for the logarithmic property prices which is free of characteristic variation of a particular property. The original unity of this average price is around R$ 568, which could be interpreted as the base price for the year 2004. Despite the marginal analysis, which is exempt from random effects, it would be interesting and important, explore the fashions estimated of the random effects intra property is that will transmit the practical utility of the fitted model.

Taking as example, the fashions of the random effects of intercept and slope for residential property label 20 are equal to 1.741 and -0.001, so you can individualize the comparison of the uncertainty of their prices compared to a second property. Another important example is the residential property labeled 1, which were estimated random effects for intercept and slope equal to -0.582 and -0.001.

The salient features of these two properties is that the label 1 showed values well below their label 20 in the whole observed period. This is reflected by the positive deviation of the property intercept 20 which is three times larger than the module of the deviation corresponding to the intercept of property 1. Moreover, price levels for the Property 1 are not only much smaller than the Property 20, but are also lower than the global average.
However, a feature common to both real estate is the decrease in the slope compared to the global deterministic slope. Moreover, such a reduction in the slope is shown more intense for the property 20.

If this behavior continues, or if the adjusted model was used in long-term projections, that would imply that the prices of these two properties would intersect in the future. It is not advisable to make medium or long-term projections using this model because there is no guarantee that the phenomenon under study maintain statistical information preserved as this period of 11 years of observations.

Short-term projections may be useful and can provide interesting results. For example, with this model (FM3) it is possible to estimate the average rents for commercial real estate to be verified in 2015. Using the estimates in Table 4, it is estimated that by 2015 the global average price of commercial real estate will be equal

\[ e^{6.343048+0.059089 \times 12 + 0.201368 \times 1} = \text{R$1.413,02}. \]

For residential properties, the estimate for 2015 is

\[ e^{6.343048+0.059089 \times 12 + 0.201368 \times 0} = \text{R$1.155,3}. \]

It is worth noting that the ratio of the predictive expression of commercial real estate and residential can be interpreted as the relative change in price when opting for the first type to replace the second. This ratio, considering the same moment in time, is equal to \( e^{0.201368} \approx 1,22 \). I.e. disregarding the individual influences of each sampled property, it is estimated that, overall, the price of renting a commercial property is, on average, 1.22 times the price of residential property rentals.

**CONCLUSIONS**

The initial objectives of the study were met, given that the study aimed to temporal modeling of prices of residential and commercial properties in the city of Mariana, Minas Gerais. The initial statistical analysis of prices, the longitudinal modeling and statistics inspections made it possible to achieve important results about the dynamics of rental prices in the city.

It was observed that: (i) projections for the rental prices deserve to be treated for each property individually, because the random effects from individual to individual is the main factor that explains the overall variability of the price series; (ii) the final model adopted (FM3)
reinforced the suspicion that the global average price of rents has a tendency to increase over time; (iii) it is estimated that no correlation exists between the levels of prices observed in 2004 and the rate at which will increase over time, and; (iv) it is estimated that the average prices of commercial real estate rents are around 1.2 times higher than the average prices of residential.

Prices for commercial rents showed a greater variability over time in relation to residential properties. It was also possible to understand that not all observed properties remained the same pricing behavior (based only on IGPM correction). Both residential and commercial properties showed speculative price behaviors over the market.

By performing a historical study in the city of Mariana and after lifting of macroeconomic factors, it can infer that both the prices of residential real estate, as commercial real estate are suffering not only the influence from rentals inflation rates but also from geographical aspects, financial and property speculation of the owners of the city's real estate.

REFERENCES


